Certainty in Mathematics

Lesson 2: Fact-Finding Task

Focus: *How do real-world controversies in mathematics reveal tensions between certainty, truth, and perspective?*

Objective	**Analyze case studies where the reliability of mathematical proofs or models was contested, limited, or debated.** **Understand how different perspectives (mathematicians, scientists, policymakers, corporations, the public) influence what counts as "true" or "reliable" mathematical knowledge.** **Apply TOK concepts (certainty, truth, perspective) to evaluate how mathematical knowledge is constructed, legitimized, or undermined.** **Substantiate or challenge claims from Lesson 1 with real-world evidence and reasoning drawn from mathematical controversies.**
Homework Preparation Task	**Case Study Task** - Divide students into small groups and assign each group a real-world mathematical controversy. Students will add their findings to the Kialo discussion from Lesson 1. - Each group will: - Reflect on how the case connects to the concepts discussed in Lesson 1 (certainty, truth, perspective). - Explore the case using provided resources and their own research. - Prepare a short presentation (5–7 minutes) responding to the question: "How does the chosen case highlight the limits of mathematical certainty and the risks of overreliance on proofs or models?" - Students should include details of: - What happened in the case. - Who had the power to decide or act (mathematicians, computers, scientists, policymakers, the public). - Which perspectives were marginalized or excluded. - Which TOK concept (certainty, truth, perspective) is most relevant. - Whether the case supports or challenges a claim from Lesson 1. **Case Study Options** **Gödel's Incompleteness Theorems (1931)** - Focus: Proved that some mathematical truths cannot be established within a system. - Key Question: Can mathematics ever be a complete and certain system of knowledge? - Suggested Sources: Stanford Encyclopedia of Philosophy – Gödel's Incompleteness Theorems; ▶ The paradox at the heart of mathematics: Gödel's Incompleteness Theorem… <ol start="2"> **Four Color Theorem (1976)** - Focus: First major computer-assisted proof; relied on machine calculations beyond human checking. - Key Question: Should we trust a proof if humans cannot verify every step? - Suggested Sources: ▶ The Four Color Map Theorem - Numberphile . <ol start="3"> **Banach–Tarski Paradox** - Focus: A mathematically valid theorem showing a ball can be split and reassembled into two identical copies. - Key Question: What happens when mathematical truth contradicts physical intuition and reality?

	- Suggested Sources: [Does math have a major flaw? - TedEd](#); ▶ The Banach–Tarski Paradox . **4. 2008 Financial Crisis (Gaussian Copula Model)** - Focus: Risk models gave a false sense of certainty, contributing to global collapse. - Key Question: What are the dangers of overreliance on mathematical models in real-world decision-making? - Suggested Sources: [Recipe for Disaster: The Formula That Killed Wall Street	WIRED](#)

Activities	**1. Introduction (5 mins)**
	- Recap Lesson 1: Review key claims from the Kialo discussion on whether mathematical structures (proofs and models) reveal truth or just create illusions of certainty.
	    ○ Prompt: Which arguments from Lesson 1 did you find most convincing or flawed? Did any claims rely too much on abstract assumptions without real-world evidence?
	- Present the central question for this lesson: *How do real-world mathematical controversies reveal tensions between certainty, truth, and perspective?*
	- Emphasize applying certainty, truth, and perspective to evaluate how mathematical proofs and models are judged by mathematicians, scientists, policymakers, and the public.
	
	2. Bridge to Lesson 2
	- Explain that in this lesson, students will explore real-world mathematical controversies where:
	    ○ Proofs or models were challenged, ignored, or revised (e.g., Gödel's Incompleteness, Four Color Theorem, 2008 crisis models).
	    ○ Perspectives from mathematicians, scientists, policymakers, corporations, and the public shaped whether the knowledge was seen as legitimate, reliable, or flawed.
	- These cases highlight that mathematical knowledge is not always neutral, universally accepted, or free from human assumptions, values, and power structures.
	- Clarify the shift: This is no longer about theory alone — students are now testing claims from Lesson 1 using historical and contemporary examples.
	- Reinforce the goal: Move from debate to evidence-based evaluation. These case studies will help students understand how mathematical knowledge is constructed, challenged, or reshaped within logical, institutional, and social frameworks.
	
	3. Presentations (30 mins)
	- Student groups present their case studies (e.g., Gödel's Incompleteness, Four Color Theorem, Banach–Tarski, 2008 Financial Crisis models, COVID-19 models, Climate Change models).
	- Students take notes on useful points from other groups to bring back into the Kialo discussion.
	
	4. Recording Findings in a Kialo Discussion (20 mins)
	- Students return to the Kialo discussion from Lesson 1 and:
	    ○ Add at least one new claim or counterclaim based on their case study.
	    ○ Reply to at least one peer's argument, using insights from another group's case.
	    ○ Label their post with the relevant TOK concept (e.g., Certainty – Gödel, Truth – Banach–Tarski, Perspective – climate models).
	
	- **Focus Areas for Kialo Updates**
	    ○ Mathematical Gatekeeping: Who decides which proofs or models are accepted as legitimate knowledge, and who gets excluded?
	    ○ Institutional Authority: How do journals, universities, corporations, or governments influence the perceived legitimacy of mathematical structures?
	    ○ Trust and Reliability: How do controversial proofs or failed models (e.g., financial risk formulas) affect public trust in mathematics?
	    ○ Knowledge Inequality: Are all mathematical voices — especially alternative systems, critical perspectives, or less powerful institutions — treated equally when defining "truth"?

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Reflection Questions	Discuss the following reflection questions in open discussion or exit ticket format: ● How did your case study affect your understanding of who gets to define and legitimize "truth" in mathematics? ● What made certain examples feel more like illusions of certainty or institutional overreach, versus genuine reliability? ● In your case, who had the most control over the mathematical narrative: mathematicians, computers, policymakers, corporations, or the public? ● Can efforts to make mathematics more rigorous (e.g., formal verification, better data in models) ever be truly equal when access to resources, funding, and authority remains uneven? ● What role should credibility, transparency, and perspective play in deciding whether mathematical structures are accepted as truth? ● Should all mathematical models that affect public wellbeing (e.g., finance, health, climate) require broader social consultation, or are there exceptions?
Resources	[Lesson Slides](#) Kialo discussions from previous lesson
TOK Concepts	**Certainty:** Who is responsible for ensuring that mathematical structures (proofs or models) are presented as certain or reliable? Should mathematicians, scientists, or policymakers carry ultimate accountability when assumptions or limitations are hidden? **Power:** How do power structures (e.g., universities publishing proofs, corporations funding models, governments using algorithms) shape which mathematical knowledge is accepted, silenced, or elevated as "truth"? **Perspective:** How do cultural, social, and historical viewpoints influence what counts as reliable mathematics? Are mathematical truths universal, or do their applications and interpretations shift depending on context?
Critical Thinking Concepts	● **Confronting Biases & Assumptions:** 　○ Challenging Neutrality Bias: Questioning the assumption that mathematics is always objective and free from human influence. 　○ Recognizing Overreliance Bias: Examining the belief that mathematical proofs and models automatically yield truth, and considering when this trust becomes misplaced. ● **Exploring Contexts:** 　○ Stakeholder Analysis: Identifying who benefits or suffers when a proof or model is accepted or rejected (e.g., mathematicians, policymakers, corporations, the public). 　○ Cultural and Social Influence: Considering how history, politics, or societal needs shape which mathematical structures are trusted or dismissed (e.g., GDP vs. alternative wellbeing measures, climate models under political scrutiny). ● **Responsiveness and Flexibility of Thought:** 　○ Adapting Arguments: Being willing to reconsider whether a proof or model is reliable after engaging with its assumptions, limitations, or failures. 　○ Comparing Conflicting Interpretations: Weighing different perspectives — mathematicians, scientists, policymakers, or media — on the legitimacy of a proof or model. ● **Extrapolation & Reapplication of Principles:** 　○ Applying Logical/Ethical Reasoning: Assessing whether reliance on a mathematical structure was fair, justified, or misleading, and considering its wider consequences. 　○ Connecting to Modern Issues: Relating lessons from past controversies (e.g., Gödel, Four Color Theorem, 2008 financial crisis models, COVID models) to current debates on AI, algorithmic bias, and predictive analytics. 　○ Building Mathematical Literacy: Reflecting on how access to data, computing power, or institutional recognition shapes who gets to decide what counts as "true" mathematics.