# Certainty in Mathematics

## Lesson 1: Opening Debate

**Focus:** *Do mathematical structures reveal truth or just create illusions of certainty?*

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| **Objectives** | • **Explore how proofs and models function as structures of certainty in mathematics.**<br>• **Debate whether mathematics provides truth or only useful approximations/assumptions.**<br>• **Apply TOK concepts of certainty, truth, and perspective to mathematics.** |
| **Activities** | **1. Introduction (5 mins)**<br>• Present the central question: "Do mathematical structures reveal truth, or just create illusions of certainty?"<br>  ○ Certainty: Are mathematical truths absolute or conditional on axioms and assumptions?<br>  ○ Truth: Do models and proofs describe reality or only provide persuasive structures?<br>  ○ Perspective: How do mathematicians, scientists, and the public differently interpret the authority of mathematics?<br>• Show a short slideshow of landmarks in logic, proof, and prediction:<br>  ○ Gödel's Incompleteness Theorems (1931): Some truths cannot be proven within a system.<br>  ○ Four Color Theorem (1976): First major computer-assisted proof — raised questions of trust and verification.<br>  ○ Banach–Tarski Paradox: Mathematically valid but physically impossible — challenges realism.<br>  ○ 2008 Financial Crisis: Reliance on risk models (Gaussian copulas) contributed to collapse.<br>  ○ COVID-19 Epidemiological Models: Predictions guided global policy but often contradicted one another.<br>  ○ Climate Change Models: Essential for policy, but attacked as uncertain or biased depending on perspective.<br>• Ask students to consider:<br>  ○ Which of these examples do you find most convincing, and why?<br>  ○ Can something be mathematically valid but still misleading?<br>  ○ Does mathematical certainty depend on proofs, models, or social trust?<br><br>**2. Debate Setup (5 mins)**<br>• Use the Kialo discussion: "*Does mathematics reveal objective truth about reality?*"<br>• Students respond to the thesis "Mathematics reveals objective truth about reality" with starter claims.<br>• Give students time to examine the starter claims, based on the points below.<br>  ○ Starter Claim 1: Mathematical proofs provide certainty and universality.<br>    ■ Support: A proven theorem (e.g., Pythagoras) is true in all possible contexts and cultures (2 + 2 = 4 everywhere).<br>    ■ Counterclaim: Proofs only hold within chosen axioms (e.g., Euclidean vs. non-Euclidean geometry).<br>    ■ Reasoning Question: Can a proof be called "truth" if it depends on assumptions?<br><br>  ○ Starter Claim 2: Mathematical models reveal truth through prediction.<br>    ■ Support: Climate models and epidemiology guide policy and save lives.<br>    ■ Counterclaim: Models simplify reality; overreliance caused failures like the 2008 financial crash.<br>    ■ Reasoning Question: Is usefulness the same as truth? |

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| | ○ Starter Claim 3: Mathematical proofs only hold within chosen axioms.<br>■ Support: Mathematical results depend on the assumptions of their systems.<br>■ Counterclaim: Within those systems, proofs still provide certainty and internal truth.<br>■ Reasoning Question: Are truths that depend on frameworks truly “objective”?<br><br>○ Starter Claim 4: The authority of numbers can mislead.<br>■ Support: Metrics like GDP or algorithmic scores can distort complex human realities.<br>■ Counterclaim:Logical grounding makes numbers more consistent than opinion.<br>■ Reasoning Question: When does mathematical authority clarify, and when does it obscure?<br><br>**3. Debate (15–20 mins)**<br>● Students present arguments and counterarguments, citing real-world cases (Gödel, Four Color Theorem, Banach–Tarski, 2008 financial crisis, COVID models, climate change).<br>● Teacher guiding prompts:<br>○ Does a proof guarantee truth, or just truth within a system?<br>○ Should we trust computer-assisted proofs the same way we trust traditional proofs?<br>○ Is predictive success enough to justify calling a model “true”?<br>○ Who should decide when a mathematical model is reliable — mathematicians, scientists, or society?<br>○ How do the TOK concepts of certainty, truth, and perspective help us evaluate these dilemmas? |
| **Reflection Questions** | Discuss the following reflection questions in open discussion or exit ticket format:<br>● Did mathematics in these cases strengthen or weaken the reliability of knowledge?<br>● Can flawed or misleading mathematical structures (proofs or models) ever be justified by their usefulness?<br>● Who should bear responsibility for ensuring mathematical models and proofs are applied responsibly — mathematicians, scientists, or policymakers?<br>● How does perspective (pure vs. applied math, scientific vs. societal, cultural vs. universal) change what is seen as mathematically “true”?<br>● How do power structures (governments, corporations, academic institutions) influence which mathematical questions are pursued, funded, or trusted? |
| **Resources** | Lesson Slides<br>Kialo Discussion: Does mathematics reveal objective truth about reality? |
| **TOK Concepts** | **Certainty:** What duties do mathematicians and model-builders have in presenting their work as certain or reliable? Should the pursuit of mathematical elegance or predictive success ever outweigh clarity about assumptions and limitations?<br>**Power:** Who holds the authority to decide what counts as legitimate mathematical knowledge — pure mathematicians, applied scientists, governments, or corporations? How do power dynamics influence which proofs are recognized and which models are trusted or implemented?<br>**Perspective:** How do cultural, historical, and societal contexts shape what is considered “true” in mathematics? Should mathematical truth be seen as universal, or does its meaning shift when applied in different cultural or practical contexts? |
| **Critical Thinking Concepts** | ● **Confronting Biases & Assumptions:**<br>○ Challenging Certainty-at-All-Costs Bias: Reflecting on the assumption that mathematical proofs and models always yield truth, and questioning who benefits when their limitations are ignored.<br>○ Recognizing Universality Bias: Examining the belief that mathematics is always universally valid, and considering what perspectives are excluded when alternative axiom systems or cultural approaches to knowledge are dismissed.<br>● **Exploring Contexts:** |

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| | ○ Stakeholder Analysis: Considering who benefits or is harmed when mathematical models or proofs are accepted as "truth" — e.g., scientists, policymakers, corporations, or the public.<br>○ Power and Political Influence: Investigating how funding, institutional authority, or political agendas shape which models are trusted, which proofs gain recognition, and how mathematical authority is used in decision-making.<br>● **Responsiveness and Flexibility of Thought:**<br>○ Adapting Judgments: Being open to rethinking whether mathematical certainty should be taken at face value after engaging with cases like Gödel's Incompleteness, the Four Color Theorem, or failed predictive models.<br>○ Comparing Frameworks of Reliability: Weighing how different approaches — formal rigor, predictive accuracy, or social trust — shape what we consider valid or reliable mathematical knowledge. |